Conceptual Design Technique Employing Graph Theoretic Duality

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Abstract : The work reported in the paper employs a general approach of associating engineering domains with general discrete mathematical models, called – graph representations. Once the engineering system is represented by a graph representation, all the reasoning processes upon the system are substituted by a reasoning of a more mathematical nature over the graph representation. Accordingly, graph representations were shown to be a unified mathematical framework for a variety of engineering domains.

One of the strongest features of graph representations is the mathematical duality relation between their different types. It was proved in the previous works that duality relations between the representations, yield relations between engineering systems belonging to similarly unrelated disciplines. These relations will be employed in this paper to establish an efficient design technique, by means of which existing engineering designs are transformed to related engineering disciplines.

The idea behind the technique introduced in the paper is first to transform the given design problem to a problem in its dual engineering domain. Then, if a solution exists in the dual domain, it is transformed back to the original engineering domain, yielding the solution to the original problem. This technique has already been applied to solve diverse design problems. The paper introduces two design cases, involving the fields of machine theory, structures and MEMS.

Keywords: Conceptual design, Graph Representations, duality, MEMS, mechanisms, micro-mirrors

1. Introduction

Duality is a well known mathematical term constituting a relation between two mathematical or physical systems. Usually, a group of mathematical entities (such as variables) of a certain type governing the behavior and defining one system is congruent to a set of different type of entities in the dual system.

In the previous publications [1-3,22] it was shown that certain engineering domains are dual to one another. This relation was systematically established through graph representations – the mathematical models representing the behavior of the systems from these domains.

For graph representations, the mathematical basis of the duality relation lies in the duality between linear graphs [4]. By definition, two graphs are dual if set of circuits of one coincides with the set of cut-sets of the other.

When considering this relation in light of specific graph representations, duality relations for specific pairs of Graph Representations are revealed. For example, in [1] two graph representations were introduced - Flow Graph Representation (FGR) and Potential Graph Representation (PGR) (see Table 1). It was then proved that for each Flow Graph Representation there exists a corresponding dual Potential Graph Representation and vice versa. The duality between the two types of representations did not imply only that their underlying graphs are dual, but also that the vector of flows of the former representation is equal to the vector of potential differences of the latter.

Fig. 1 depicts this type of relation between the engineering systems.

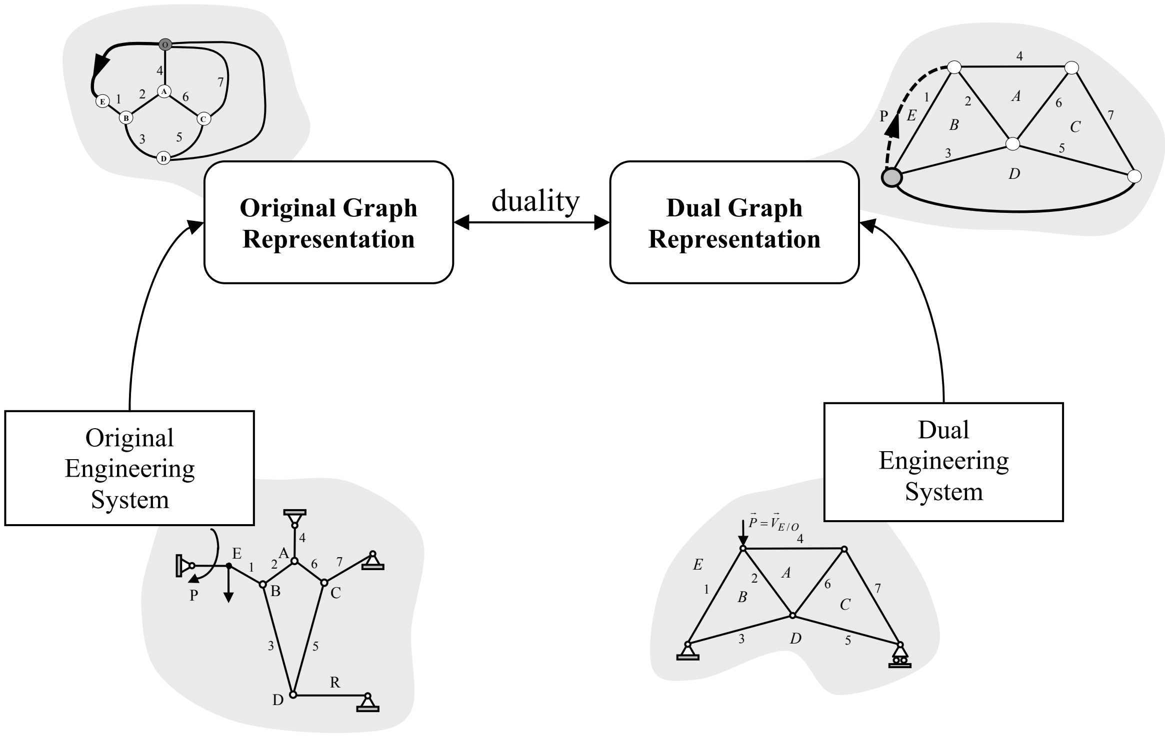


Fig. 1 Duality relation between mechanisms and trusses

The design technique introduced in this paper employs the duality relations in order to establish knowledge transfer channels between the corresponding engineering domains. Such channels would enable one to transfer designs that are already known in one engineering field to new designs in the other.

Several aspects underlying the approach suggested in the current paper relate to well-known topics of the contemporary engineering design research. Following is a brief literature review on these topics, including 'design by analogy', 'systematic design', 'representation of designs' and others, provided to enable the reader to compare these topics to the current work.

The new designs obtained by means of the approach suggested in this paper are mathematically isomorphic to already known designs in other engineering fields. Because of that, there is a certain correlation between the techniques proposed in this paper and the 'design by analogy' approach that has recently gained a wide appreciation among the engineering design and AI communities.

Balazs and Brown [5], for example, used analogical reasoning for simplifying a design so to reduce the computation complexity. A computational theory of analogy-based creative design called 'model based analogy'

(MBA) was developed by Goel [6] , who used models to represent explicitly the structural elements of a device, its topology and the function. Unlike the suggested approach, where mathematical foundation of graph theory underlies the process of deriving the design, a majority of the works in the field of design by analogy attempt to simulate the process of how the designers and engineers arrive at their solutions. Particularly, in this regard, one should mention the works of Gero dealing with situated analogy in design [7].

Additional correspondence to the approach reported in this paper and the field of analogy based design can be traced in works that also employ topological diagrams and graphs. For example, Borner et al. created a library of design concepts that express topological patterns, and employed the best matching algorithm to retrieve an appropriate design candidate [8]. Another approach that used graphs to restore designs was performed by Qian and Gero [9] who represented designs in the form of a function-behavior–structure model.

The approach suggested here enables deriving systematically new engineering designs [21]. The systematicity of the suggested techniques follows from the mathematical basis underlying graph representations, which gives rise to deterministic rules for treatment of engineering systems.

A known approach, called TRIZ, for systematic design, was developed by Altshuller [10]. The principles of TRIZ were developed upon investigating thousands of existing inventions and patents, while in this approach the investigation was performed upon the representations and their interrelations.

Another direction in design was reported by Urlich and Seering [11] who employed the schematic description describing the topology of engineering systems to produce new engineering designs. Given a design problem in a form of a function of input-output relation, they generated initial 'candidate' systems, constructed the corresponding 'compact descriptions' and applied modifications upon them to adjust the system behavior to the problem requirements. In the approach presented in the paper the design is performed upon the graph representations, that not only constitute a schematic description of the systems, but incorporate additional inherent properties, such as behavior, that are employed in the design process.

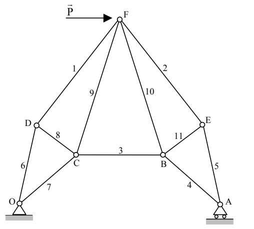
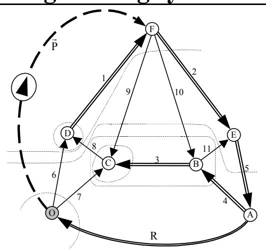
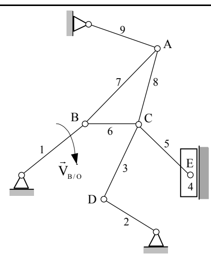
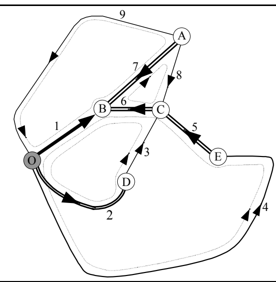
The issue of representing the design specification is widely dealt in design community, so that the representation possesses both the geometry and the behavior of the design. One of the works related to this issue was carried out by Finger and Rinderle [12] where the bond graphs [13] were used as a representation. In the current paper the behavior of the system is inherent in the representation, and can be derived upon applying the representation rules. This issue makes the representations more compact and more convenient to reason over.

2. Graph Representations

Graph Representation is an isomorphic graph-theoretical substitute of an engineering system, the embedded mathematical knowledge of which is used to map the systems' behavior. The purely mathematical essence of Graph Representations makes them convenient for computerization and enables them to provide tools for a generalized treatment of the systems. Different types of Graph Representations are characterized by four main parts: embedded knowledge, relations to other graph representations, represented engineering domains and rules for construction of the representation. In more detail, the description of the aspects characterizing graph representations can be found in [14,20].

Till now, several types of graph representations were reported and employed to represent different engineering domains [14]. Current paper utilizes two of the representations: flow graph representation (FGR) and potential graph representation (PGR), the basic properties of which are summarized in Table 1.. For more details on these representations, the reader is referred to publications listed in the last column of the table.

Table 1. Graph Representations developed in previous publications

Type of graph representation	General Description	Related engineering disciplines	Example of engineering system	Representation of the example engineering system	References
Flow Graph Representation (FGR).	Each edge in FGR is associated a vector called 'flow'. Flows in FGR satisfy the "flow law", stating that sum of flows in each cut-set is equal to zero.	Determinate structures, static systems, electric circuits.			[1,2,14]
Potential Graph Representation (PGR).	Each vertex in PGR is associated a vector, called 'potential' that satisfies the 'potential law', saying that the sum of potential differences in each circuit is equal to zero.	Mechanisms, gear trains, electric circuits.			[1,3,14]

In addition to the information provided in Table 1, an important property of these two representations is that they are related to one another through the duality connection. It was proven in [1] that for each flow graph representation there exists a dual potential graph representation, which satisfies: *the graphs underlying the two representations are dual [4], the flows in the edges of the former are equal to the potential differences in the corresponding edges of the latter.*

As was shown in [1] - the duality relation between the representations leads to establishing of a duality relation between the represented engineering systems, which yielded a variety of practical engineering applications.

3. Design through the duality relation

Current section introduces a general technique for employing the duality relation between engineering systems for design and demonstrates it on two practical examples. As was explained above, the essence of the approach adopted in this paper is to obtain a new engineering design by transferring a known one from some other field through mathematical relations established earlier.

When facing a specific engineering design problem, the important issue to be resolved prior to commencing above process is to decide what known engineering system from other engineering domain should be transferred. This decision is made in the same systematic manner as the process of devising of new design, but the process is done in the opposite direction: the problem formulation is transferred from the domain in which the engineering system is to be found to the second domain. Then, it is checked what known engineering system satisfies the obtained requirements and if such system is found it is transferred to the original engineering domain. Following is the algorithmic description of the technique:

The dual graph design technique.

1. Originally the requirements from the engineering system design are formulated in the terminology of the relevant engineering domain (*original engineering domain*).
2. The problem statement is translated into the terminology of the corresponding graph representation (*original graph representation*), and becomes a problem in the representation.
3. The problem statement obtained in step 2 is translated through the duality relation to the terminology of the dual graph representation (*secondary graph representation)*.
4. The problem statement obtained in step 3 is translated to the terminology of the second engineering domain that is represented by the dual graph representation.
5. The problem is solved in the *secondary engineering domain*.
6. The graph of the engineering system obtained in step 5 is built. Algorithms for constructing representations of engineering systems are described in corresponding publications (Table 1).
7. The graph representation dual to the graph obtained in step 6 is built thus the representation for the original design problem is obtained.
8. From the graph obtained in step 7, an engineering system from the *original engineering domain* is built. The construction process can be performed gradually, by augmenting one element of the system at a time.

Fig. 2 presents the flowchart describing the above design process.

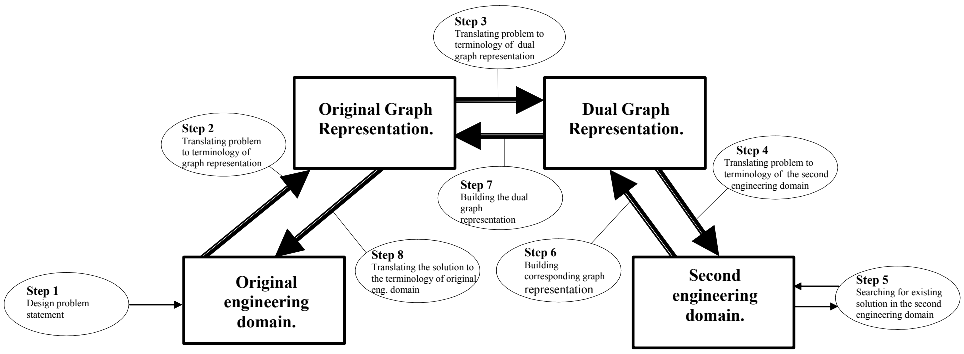


Fig. 2 Flowchart outlining the dual design technique

3.1 Design by means of the duality relation between mechanisms and determinate trusses

Flow Graph Representation is used to represent determinate trusses [1] and Potential Graph Representation is used to represent mechanisms [1], thus we can establish a knowledge transfer channel between the two systems passing through the duality relation between their representations.

This channel makes possible designing new trusses, starting from known mechanisms, or conversely - new mechanisms starting from known trusses.

The terms of dual design technique for such a case are listed in Table 2.

Table 2. Correspondence between the terminology of dual graph design technique and the case study

Dual graph technique	Current example
Original engineering domain.	Trusses.
Secondary engineering domain.	Mechanisms.
Original graph representation.	Flow Graph Representation.
Secondary (dual) graph representation.	Potential Graph Representation.

The correspondence between the terminologies of the graph representations and the two engineering domains, as was established in [1] is briefly described in Table 3..

Table 3. FGR and PGR construction rules and the duality relation between them

Terminology of the original engineering domain (trusses)	Terminology of the original graph representation (Flow Graph Representation)	Terminology in the secondary representation (Potential Graph Representation)	Terminology in secondary engineering domain (mechanisms)
Truss element (rod, external force, reaction).	Edge.	Edge.	Link, slider.
Area closed by rods.	Face.	Vertex.	Kinematical pair.
Internal force of the element.	Flow through the edge.	Potential difference of the edge.	Relative velocity of the link.
	Cut-set.	Circuit	

Following is an example of applying the technique for solution of a specific truss design problem.
Following four steps deal with transferring the problem formulation from trusses into the terminology of graph representations and then to mechanisms. This transfer process is schematically outlined in Fig. 3.

Step 1. Stating the Design problem in the terminology of the original domain
Design a determinate truss obtaining a finite external load as an input and returning a very large internal force in a specific rod as an output.

Step 2. Transforming the design problem into the terminology of the original graph
Formulating the problem in the Flow Graph Representation terminology yields: find a flow graph getting a finite flow in the flow source, which is then highly amplified in some other specified edge.

Step 3. Translating the problem to Dual Graph Representation terminology**:** find a potential graph representation getting a finite potential difference in the potential difference source edge and a highly amplified potential difference at the specified edge.

Step 4. Translating problem statement from dual graph to terminology of the secondary engineering domain. Translating the problem from PGR to mechanisms: find a mechanism receiving a finite velocity at the driving link and highly amplified velocity at the output.

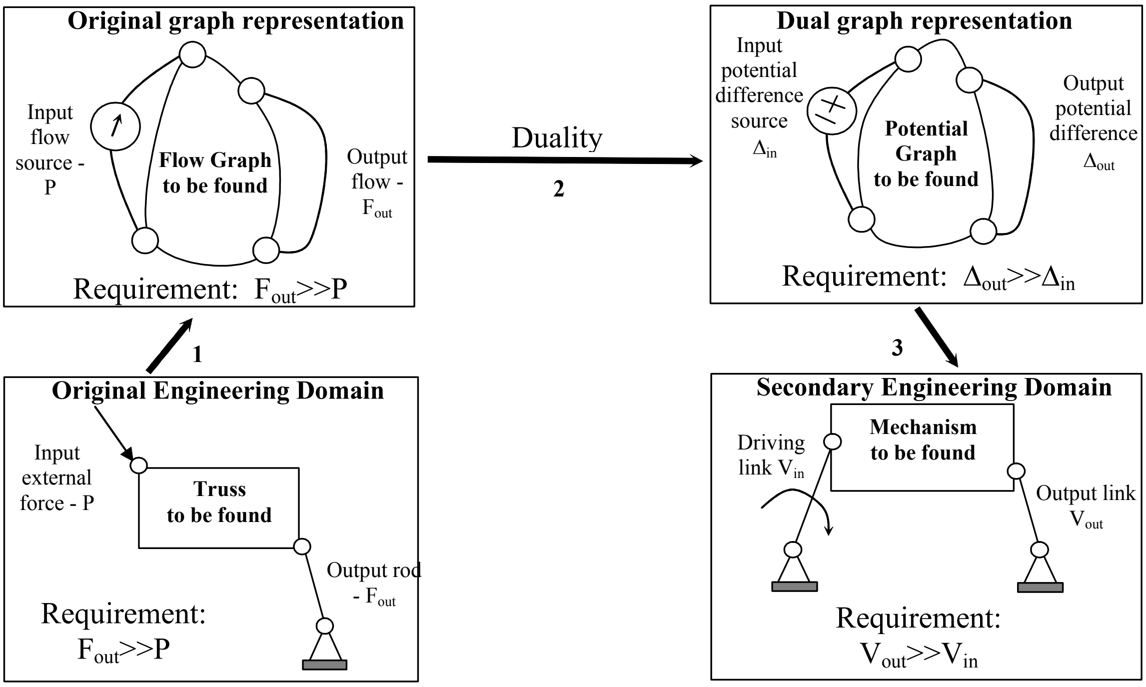


Fig. 3 The transformation process from the truss to mechanism problem

Step 5. Solving the problem in the secondary domain. The solution for a mechanism design problem, as it is stated in step 3, can be obtained in a straightforward manner through employing instant center method from machine theory [15], as is shown in Fig. 4.

Finally, the design of the mechanism can be translated through the graph representations into a new design of a truss. Steps 5-7 for obtaining the truss design complying to the original requirements are shown in Fig. 5.

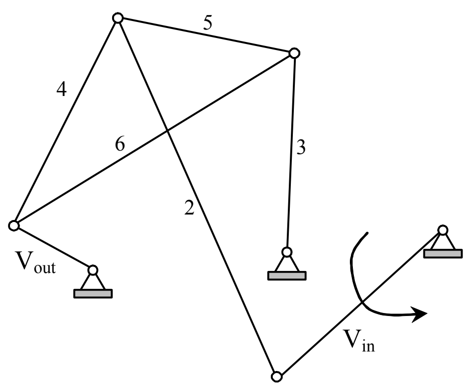


Fig. 1 Solution for the mechanism design problem

Step 6. Constructing the graph for the design solution obtained in the secondary engineering domain. The potential graph corresponding to a mechanism is constructed by means of the construction rules given in Table 3**.**.

Step 7. Constructing graph dual to the graph obtained in step 6. A flow graph is obtained from the potential graph corresponding to the mechanism in the secondary domain by means of graph theory duality construction rules (Table 3).

Step 8. Building an engineering system for the original engineering design from the graph obtained in Step 7. In this step the design solution is transformed from the graph representation to the original engineering domain. In this design case a truss is constructed from the flow graph obtained in Step 6.

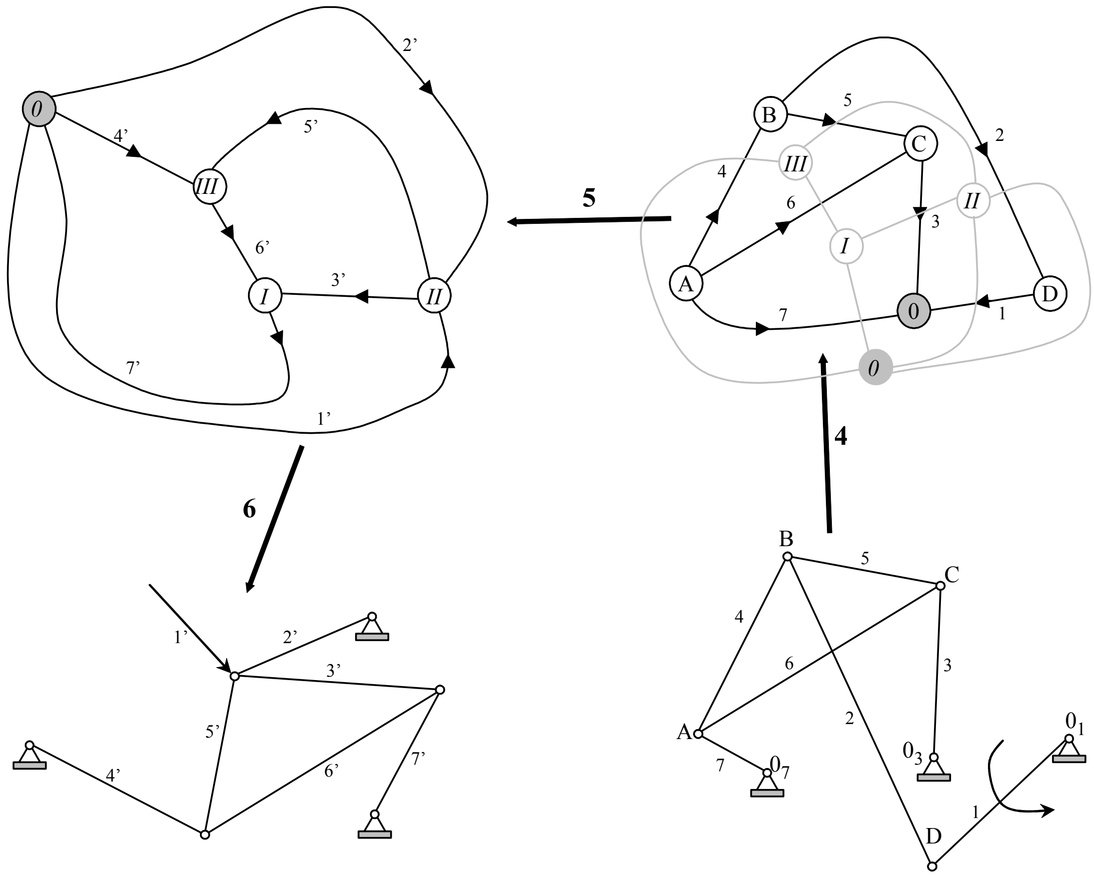


Fig. 5 Obtaining a new truss design from the known design of a mechanism

4. Employing the duality technique in MEMS

Current section demonstrates application of the dual design technique to design of a micormirror. In general a micromirror is composed of a mirror of microscopic size and a suspension device capable of inclining the mirror to an arbitrary spatial angle. In order to facilitate the explanation no further technical specifications would be considered, thus the design would be carried out on rather conceptual than specific level.

In the modern industry, the production of MEMS devices involves certain limitations [16,17], which impose additional constraints on the desired design. The most basic constraints that should be taken into account in this example are the following:

1) The motion in devices of this type is produced by various actuators, such as electrostatic, electromagnetic, thermal or piezoelectric. Usually, due to the very small scales of the device, it is hard to model the displacements made by the actuators, but it is much easier to model the forces that they exert [18]. Here, this constraint can be fulfilled by a proper choice of the graph representation for the original design problem. Specifically, as appears in Table 1 the flow graph representation (FGR) – accounts only for the forces acting in the system, while disregarding the displacements. Consequently this representation is most appropriate for representing the problem of micromirror design.
2) Due to the nature of the construction process [17], all the mechanical elements of the micro-devices usually constitute a single monolithic entity. This requirement is to be taken into account during the final design stage – when constructing the final design from the corresponding graph.

Following are the steps of the duality design process applied to this problem:

Step 1. Stating the Design problem in the terminology of the original domain

Design a micro system containing a mirror found in force equilibrium and acted upon by two independent moments – one causing it to rotate around the x axis and the other around the y axis.

Step 2. Transforming the design problem into the terminology of the original graph

Formulating the problem in the Flow Graph Representation terminology yields: find a two-dimensional flow graph, where a specific edge possesses two independent flows – one possessing only x component and other only the y component.

Step 3. Translating the problem to Dual Graph Representation terminology.

The representation dual to the flow graph representation is the potential graph representation. Translating the problem statement of Step 2 to the terminology of PGR yields: find a two-dimensional potential graph, where specific edge is forced to possess potential difference containing two independent orthogonal components – one directed in the x axis and the other in the y axis.

Step 4. Translating problem statement from the dual graph to terminology of the secondary engineering domain.

Since the behavior of the original system is based on rotational variables (moments), it is preferable to seek the same in the dual system. One of the engineering domains that can be represented by the potential graph representation is the domain of gear trains, thus this domain will be chosen to be the dual domain. It is interesting to notice that while original engineering domain belonged to micro-systems, the dual domain is actually a macro-domain. This issue provides the system designer with a significant advantage, since in contrast to micro-systems, the field of macro-systems has been developed for hundreds of years and possesses much more accumulated knowledge.

In the terminology of gear trains the problem becomes: Design a gear train with an element being rotated around two axes – the x axis and the y axis.

Step 5. Solving the problem in the secondary domain.

One of the trivial solutions to this problem that is likely to be suggested by an expert in the field of machine theory is shown in Fig. 6.

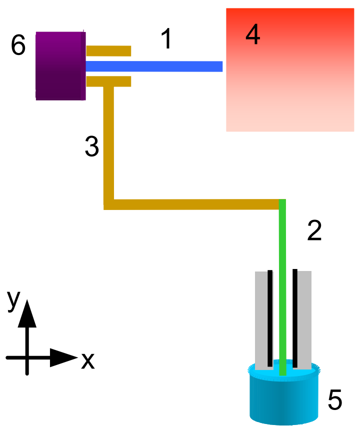


Fig. 6 Gear train satisfying the requirements of Step 4

The principle of the device shown in Fig. 6 is as follows: motor 5 mounted to the ground rotates shaft 2 connected with carrier 3 around the y axis. Motor 6 mounted to the carrier and rotates shaft 1 connected to the element 4. As element 4 rotates around both axes, while the rotation around the y axis is determined by motor 5 and the rotation around the x axis is determined by motor 6. Consequently the device fulfills the requirements stated in step 5.

Step 6. Constructing the graph for the design solution obtained in the secondary engineering domain.
The potential graph representation of the device obtained in the previous steps is shown in Fig. 7. The basic construction rules for the representation that were used as they appear in [1] are: assign an edge to each element of the system, while the motors are associated with potential difference source edges. The vertices of the system correspond to interconnections between the elements. The potential differences of the edges are equal to the relative angular velocities of the elements.

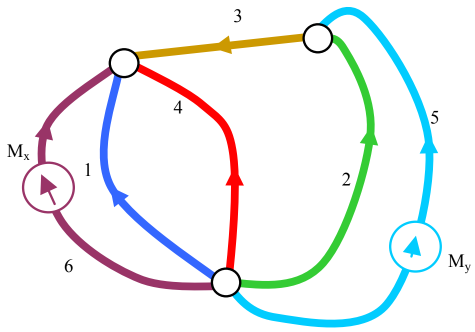


Fig. 7 The graph representation of the gear train obtained in Step 5

Step 7. Constructing graph dual to the graph obtained in step 6.
A flow graph is obtained from the potential graph corresponding to the gear train obtained in Step 6 by means of graph theory duality construction rules [2], as is shown in Figure 8.

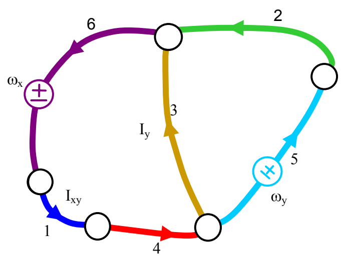


Fig. 8 Flow graph representation dual to the representation of Figure 7

Step 8. Constructing the micromirror from the graph obtained in Step 7.

According to the graph construction rules, the graph shown in Figure 8 can be interpreted as the following engineering system (Figure 9).

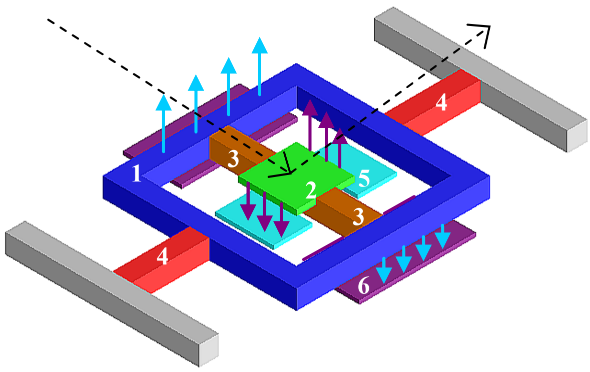


Fig. 9 Design of micromirror obtained through the duality design process

Figure 10 depicts the above process of deriving the micromirror design from the macro kinematical system through the corresponding dual graph representations.

The final design shown in Figure 9 is actually a design that is already known in the MEMS community [19]. This provides us with further confirmation on validity of the suggested design technique.

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